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Majorana Mass of the Electron–Muon Dirac Neutrino and the Fermion Masses

Cvavb Chandra Raju[1]

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The left-handed electron and muon neutrinos are considered to be Majorana neutrinos with equal mass. They have opposite *CP* parities and are equivalent to a single Dirac neutrino. These neutrinos are shown to have a Majorana mass of about 6.5 eV. The relatively large mass of their charged leptons is due to their γ_5 coupling with the Higgs scalars. By expressing the Higgs scalars as Clebsch–Gordon type of combinations of Z and D neutral vector bosons with appropriate quantum numbers, it is shown that $2m_e m_\mu/(m_e^2 + m_\mu^2) = (g_v/g_A)^2_{e\mu}$, where g_v and g_A are the vector and axial vector coupling constants, respectively, of Z (or D) with the leptons e and μ. Weinberg mixing parameters $x_L = e^2/g_L^2$ and $x_R = e^2/g_R^2$ are determined to be 0.2254 and 0.2746, respectively. In the quark sector the Cabibbo angle is about 13°11′ and the masses of t and b quarks are found to be respectively 134.2 and 4.69 GeV.

1. INTRODUCTION

The nature of weak interactions appears to be intimately connected with properties of the neutrino. When V–A theory was proposed by Sudarshan and Marshak (Marshak and Sudarshan, 1958) they based their arguments on the assumption that neutrinos are massless. The standard model (Weinberg, 1967) of electroweak interactions provides a sound mathematical basis for the V–A theory. Mohapatra and Senjanovic attribute the smallness of neutrino mass to the suppression of V + A currents (Mohapatra and Senjanovic, 1980; Mohapatra, 1981).

In this paper we show that ν_{eL} and $\nu_{\mu L}$ are Majorana neutrinos with equal mass and opposite *CP* parities. These two Majorana neutrinos are equivalent to a single Dirac neutrino. The mass of the neutrino is computed to be about 6.5 eV. The crucial ingredient responsible for our result is the

[1]Department of Physics, Nizam College (O.U.), Hyderabad—500 001, India.

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presumption that the Higgs scalars that generate masses are Clebsch–Gordon type combinations of Z and D bosons, with appropriate quantum numbers. Here Z and D are the neutral bosons of the $SU(2)_L \times SU(2)_R \times U(1)$ triangle anomaly-free weak interaction with two neutral currents (Raju, 1985, 1986a).

The aim of this paper is to find the masses of almost all fermions. Our considerations are based mainly on the following:

(1) If Z (or D) is coupled to the vector or axial vector currents of the charged particles with the coupling constants $(g_V)_q$ and $(g_A)_q$, then the coupling constants of the Higgs scalars with $\bar{q}q$ or $\bar{q}\gamma_5\, q$ for the generation of the mass of q must, respectively, be proportional to $(g_V)^2$ and $(g_A)_q{}^2$, where q is the charged fermion. To justify it we propose that Higgs scalars are Clebsch–Gordon type combinations of Z and D with appropriate quantum numbers.

(ii) The neutrinos ν_{eL} and $\nu_{\mu L}$ are Majorana-type neutrinos with equal mass and opposite CP parities. They conserve Zeldovich, Konopinsky, and Mahmoud (ZKM) lepton charge, according to which one lepton charge, the same for e^- and μ^+, is conserved (Zeldovich, 1952; Konopinsky and Mahmoud, 1953).

(iii) The results should be extendable to other fermions.

The paper is organized as follows: In Section 2 we summarize the main features of Dirac and Majorana mass terms. Section 3 presents the ZKM scheme. In Section 4 we determine the exact expression for the electron and muon masses. In Section 5 we relate the mass-determining constants to the vector and axial vector current coupling constants of Z (or D). In this section the neutrino mass and Weinberg mixing parameters are evaluated. In Section 6 the Cabibbo angle and the masses of the t and b quarks are determined. Section 7 contains a summary of our results.

2. DIRAC AND MAJORANA MASS TERMS

In a theory containing both left-handed (LH) and right-handed (RH) neutrinos of a given flavor, the neutrino fields are expressible as $\psi^{(\nu)}_{L(R)} = \Gamma_{L(R)}\psi^{(\nu)}$, where the projectors $\Gamma_{L(R)} = \frac{1}{2}(1 \pm \gamma_5)$ ensure that the massless fields $\psi^{(\nu)}_{L(R)}$ have just two components. It is possible to construct a Hermitian, Lorentz-invariant, lepton-conserving interaction which couples the two chiralities,

$$\begin{aligned} L_{\text{Dirac}} &= -m_D[\bar{\psi}^{(\nu)}_L \psi^{(\nu)}_R + \bar{\psi}^{(\nu)}_R \psi^{(\nu)}_L] \\ &= -m_D \bar{\psi}^{(\nu)} \psi^{(\nu)} \end{aligned} \tag{2.1}$$

where the field $\psi^{(\nu)}$ is now a four-component entity. The lepton conservation is ensured by the invariance of L_{Dirac} under the field transformations $\psi_L \to e^{i\Lambda}\psi_L$ and $\psi_R \to e^{i\Lambda}\psi_R$. Such an interaction is said to give rise to a

Dirac contribution to fermion mass and in the standard model it is this mechanism (via the Higgs effect) which gives rise to fermion masses. Of course, if the field $\psi_R^{(\nu)}$ does not exist, then no such construction is possible. This is why neutrinos are massless in the standard model.

A logically independent source of neutrino mass called a Majorana contribution is available even in the absence of right-handed neutrinos. Let ψ_L and ψ_R be LH and RH fields. In addition, let us consider

$$(\psi_L)^c = C\bar{\psi}_L^T, \qquad (\psi_R)^c = C\bar{\psi}_R^T, \tag{2.2}$$

where C satisfies the conditions

$$C\gamma_\alpha^T C^{-1} = -\gamma_\alpha, \qquad C^+C = 1, \qquad C^T = -C, \qquad C^{-1}\gamma_5 C = \gamma_5^T. \tag{2.3}$$

It follows from (2.2) and (2.3) that

$$(\bar{\psi}_L)^c = -\psi_L^T C^{-1}, \qquad (\bar{\psi}_R)^c = -\psi_R^T C^{-1}. \tag{2.4}$$

The field $(\psi_{L(R)})^C$ transforms as $(\psi_{R(L)})$ under the proper Lorentz transformations. It is not difficult to show that $(\psi_L)^C$ is a RH field, while $(\psi_R)^C$ is a LH field (Bilenkey and Petcov, 1987). The Majorana mass term is

$$L_{\text{Majorana}} = -m_M[\bar{\psi}_L^c \psi_L + \text{H.c.}]. \tag{2.5}$$

Although it is Lorentz invariant and Hermitian, it does not conserve lepton number. The mass term was first considered by Gribov and Pontecorovo (see Bilenkey and Petcov, 1987).

3. THE ZKM SCHEME

Let

$$\nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \end{pmatrix}, \tag{3.1}$$

where ν_{eL} and $\nu_{\mu L}$ are, respectively, the LH neutrino fields corresponding to the electon and muon. The existing experimental data are not incompatible with the assumption that to the charged leptons (e.g., e and μ) there corresponds one four-component neutrino whose LH and RH components enter into weak lepton currents. We have in mind the scheme of Zeldovich, Konopinsky, and Mahmoud (ZKM), according to which one lepton charge, the same for e^- and μ^+, is conserved. The charged lepton current has in the ZKM scheme the form,

$$j_\alpha^+ = 2(\bar{\nu}_L \gamma_\alpha e_L + \bar{\nu}_L^c \gamma_\alpha \mu_L), \tag{3.2}$$

where $\nu_L^c = \frac{1}{2}(1 + \gamma_5)\, \nu^c$ and $\nu^c = C\bar{\nu}^T$.

The electron and muon neutrinos are, in the ZKM scheme, LH neutrino and LH antineutrino, respectively. It can be shown that the CP parities the Majorana particles can assume are $\pm i$, and that the relative CP parities of Majorana neutrinos are observable quantities. In the present case let

$$L_{\text{Majorana}} = -\tfrac{1}{2}(\bar{\nu}_{\text{L}})^{c} M \nu_{\text{L}} + \text{H.c.}, \tag{3.3}$$

where M is a symmetric 2×2 matrix. We shall assume here that the CP invariance holds in the leptonic sector. One has then $M^{*} = M$. Let

$$M = \begin{pmatrix} a_1 & b \\ b & a_2 \end{pmatrix}, \tag{3.4}$$

where a_1, b, and a_2 are real parameters. Then (3.3) can be written as

$$\begin{aligned} L_{\text{Majorana}} = {} & -\tfrac{1}{2}\{a_1(\bar{\nu}_{e\text{L}})^{c}(\nu_{e\text{L}}) + a_2(\bar{\nu}_{\mu\text{L}})^{c}(\nu_{\mu\text{L}}) \\ & + b[(\overline{\nu_{e\text{L}}})^{c}(\nu_{\mu\text{L}}) + (\overline{\nu_{\mu\text{L}}})^{c}(\nu_{e\text{L}})]\} + \text{H.c.} \end{aligned} \tag{3.5}$$

Let us consider this Majorana mass term, with

$$a_1 = a_2 = 0 \qquad \text{and} \quad b > 0 \tag{3.6}$$

It is clear that the masses of the Majorana neutrinos coincide,

$$m_{1,2} = b = m, \tag{3.7}$$

while their CP parities are opposite,

$$\eta_{CP}(\chi_1) = i; \qquad \eta_{CP}(\chi_2) = -i. \tag{3.8}$$

When the conditions (3.7) and (3.8) are fulfilled, the Majorana mass term is reduced to a Dirac mass term. We then have

$$L_M = -\tfrac{1}{2} m[(\overline{\nu_{\mu\text{L}}})^{c} \nu_{e\text{L}} + (\nu_{e\text{L}})^{c} \nu_{\mu\text{L}}] + \text{H.c.} \tag{3.9}$$

The Lagrangian (3.9) is invariant with respect to the gauge transformations

$$\begin{aligned} \nu_{e\text{L}}(x) &\to \nu'_{e\text{L}}(x) = e^{i\Lambda} \nu_{e\text{L}}(x), \\ \nu_{\mu\text{L}}(x) &\to \nu'_{\mu\text{L}}(x) = e^{-i\Lambda} \nu_{\mu\text{L}}(x), \end{aligned} \tag{3.10}$$

where Λ is a constant parameter. This invariance implies that the mass term (3.9) is a Dirac mass term. Indeed, let us introduce the field $\nu(x)$ so that

$$\nu_{\text{L}} = \nu_{e\text{L}}; \qquad \nu_{\text{R}} = (\nu_{\mu\text{L}})^{c}. \tag{3.11}$$

From equations (3.9) and (3.11) we obtain,

$$L_M = -m\bar{\nu}\nu, \tag{3.12}$$

where $\nu(x)$ is a four-component Dirac field. If simultaneously we perform the following transformations of the electron and muon fields,

$$e(x) \to e'(x) = e^{i\Lambda} e(x),$$
$$\mu(x) \to \mu'(x) = e^{-i\Lambda} \mu(x), \tag{3.13}$$

the total Lagrangian of the system obviously will not change. This invariance implies that the lepton charge L', equal to $+1$ for e^- and ν_e and to -1 for μ^- and ν_μ (and to zero for all other particles), is conserved.

4. ELECTRON–MUON MASSES

In the standard model, the masses are generated by spontaneous symmetry breaking. In this case, the simplest form of the Higgs mechanism requires a neutral particle called the Higgs boson to exist. So far the Higgs boson has not been found. Suppose the electroweak model is based on the gauge group $SU(2) \times SU(2)_R \times U(1)$. In that case we will have two neutral currents. In addition to the standard Z boson, we will have one more neutral boson known as the D boson (one may call them Z_1 and Z_2). In this gauge model we need at least two Higgs scalars. Let these Higgs bosons be ϕ_L and ϕ_R corresponding to the $SU(2)_L \times SU(2)_R \times U(1)$, with

$$\phi_L = \phi_L' + V_L, \qquad \phi_R = \phi_R' + V_R, \qquad \text{and} \qquad \langle\phi_L'\rangle = \langle\phi_R'\rangle = 0. \tag{4.1}$$

In addition to these let there be a Higgs quartet ϕ_0 such that

$$\langle\phi_0\rangle = \begin{pmatrix} u_3 & 0 \\ 0 & v_3 \end{pmatrix}, \tag{4.2}$$

where u_3, v_3, V_L, and V_R are real VEVs. The Lagrangian is given by

$$-L_1 = h_3(\overline{\nu}_e, \overline{e})\left(\frac{1-\tau_3}{2}\right)\tilde{\phi}_0\begin{pmatrix}\nu_e \\ e\end{pmatrix} - h_3(\overline{\nu}_e, \overline{e})\left(\frac{1+\tau_3}{2}\right)\phi_0\begin{pmatrix}\nu_e \\ e\end{pmatrix}$$
$$+ h_3'(\overline{\nu}_e, \overline{e})\left(\frac{\tau_1 + i\tau_2}{2}\right)\phi_0\begin{pmatrix}\nu_e \\ e\end{pmatrix} + h_3'(\overline{\nu}_e, \overline{e})\phi_0\left(\frac{\tau_1 - i\tau_2}{2}\right)\begin{pmatrix}\nu_e \\ e\end{pmatrix}. \tag{4.3}$$

In (4.3) h_3 and h_3' are real and $\tilde{\phi}_0 = \tau_2\phi_0\tau_2$. The τ's are Pauli matrices. Equation (4.3) can be summarized with the following mass matrix:

$$-L_1 = (\overline{\nu}_e, \overline{e})M\begin{pmatrix}\nu_e \\ e\end{pmatrix}, \qquad \text{where} \quad M = \begin{pmatrix} -h_3u_3 & h_3'v_3 \\ h_3'v_3 & h_3u_3 \end{pmatrix}.$$

The matrix MM^{+} is automatically diagonal. Until now ν_e and e, which are mass eigenstates, have equal mass m, where,

$$m = (h_3^2 u_3^2 + h_3'^2 v_3^2)^{1/2}. \tag{4.4}$$

We use an exactly identical Lagrangian for the muon and its neutrino; then they will have the same mass m. In view of (3.11),

$$m(\bar{\nu}_e \nu_e + \bar{\nu}_\mu \nu_\mu) \equiv m[(\overline{\nu_{\mu L}})^c \nu_{eL} + (\overline{\nu_{eL}})^c \nu_{\mu L} + \text{H.c.}].$$

The charged leptons e and μ are coupled to ϕ_L and ϕ_R and the neutrinos have no coupling whatsoever with these scalars. This part of the Lagrangian is given by

$$L_2 = -[ia_L \bar{e}\gamma_5 e \phi_L + ia_R \bar{e}\gamma_5 e \phi_R + ib_L \bar{\mu}\gamma_5 \mu \phi_L + ib_R \bar{\mu}\gamma_5 \mu \phi_R], \tag{4.5}$$

where a_L, b_L, a_R, and b_R are real constants. We add L_2 to L_1. Let us first examine the electron mass part of (4.5) along with (4.3) after spontaneous symmetry breaking. The electron part is

$$-m\bar{e}e - ia_L \bar{e}\gamma_5 e V_L - ia_R \bar{e}\gamma_5 e V_R - ia_L \bar{e}\gamma_5 \phi_L' - ia_R \bar{e}\gamma_5 e \phi_R', \tag{4.6}$$

where the very first term is the contribution of (4.3).

Given a Dirac field, say ψ, the Hermitian scalar and pseudoscalar $\bar{\psi}\psi$ and $i\bar{\psi}\gamma_5\psi$ have opposite CP and T transformation properties. (In this respect they are unlike the vector and axial vector.) The CP violation is now caused by the exchange of $\phi_{(LR)}'$ fields. Since the coupling of Higgs fields is usually rather small, it is possible to arrange for the CP violation to be of roughly milliweak magnitude (Bailen and Love, 1974; Mohapatra, 1981). Let

$$e = \exp(-\tfrac{1}{2} i \alpha_1 \gamma_5) e', \tag{4.7}$$

where α_1 is a real parameter. Vector or axial vector interactions are unaffected by this transformation. We choose α_1 in such a way that the constant coefficient of $\bar{e}'\gamma_5 e'$ is zero. Thus, (4.6) gives

$$\begin{aligned} &- [m \cos \alpha_1 + (a_L V_L + a_R V_R) \sin \alpha_1] \bar{e}' e' \\ &- [-im \sin \alpha_1 + i(a_L V_L + a_R V_R) \cos \alpha_1] \bar{e}' \gamma_5 e' \\ &- a_L \bar{e}' (\sin \alpha_1 + i\gamma_5 \cos \alpha_1) e' \phi_L' \\ &- a_R \bar{e}' (\sin \alpha_1 + i\gamma_5 \cos \alpha_1) e' \phi_R', \end{aligned} \tag{4.8}$$

and we set the coefficient of the second term to zero to yield,

$$\tan \alpha_1 = \frac{(a_L V_L + a_R V_R)}{m}. \tag{4.9}$$

Using (4.9) in the very first line, we note that,

$$m_e^2 = m^2 + (a_L V_L + a_R V_R)^2 = m^2 \sec^2 \alpha_1. \tag{4.10}$$

Without any loss of generality we now define,

$$a_0 V_L = a_L V_L \left(1 + \frac{a_R V_R}{a_L V_L}\right),$$

where

$$a_0 = a_L \left(1 + \frac{a_R V_R}{a_L V_L}\right). \tag{4.11}$$

If one wishes, one may factorize $a_R V_R$ such that, in place of M_{WL}, instead M_{WR} (W_R-boson mass) appears in the ultimate expression. The mass of the electron is now given by (Raju, 1985, 1986a–c, 1987)

$$m_e^2 = m^2 + a_0^2 V_L^2. \tag{4.12}$$

From our analysis it is clear m represents the Majorana mass of the neutrinos, which is quite small. So (4.12) shows that m_e^2 is proportional to M_{WL}^2 because of V_L^2. If V_R is factorized out in (4.11), m_e^2 can be shown to be proportional to M_{WR}^2. In an exactly similar fashion to the case of the electron, (4.5) leads in the case of the muon to,

$$m_\mu^2 = m^2 + b_0^2 V_L^2, \tag{4.13}$$

where

$$b_0 = b_L \left(1 + \frac{b_R V_R}{b_L V_L}\right), \tag{4.14}$$

with

$$\mu = \exp(-\tfrac{1}{2} i \alpha_2 \gamma_5) \mu', \tag{4.15}$$

$$\tan \alpha_2 = (b_L V_L + b_R V_R)/m. \tag{4.16}$$

In (4.15) and (4.13) $a_0 \neq b_0$ since m_e^2 is not equal to m_μ^2. We may arrange it such that $a_0^2 V_L^2$ and $b_0^2 V_L^2$ contribute a term $-m^2$ in (4.12) and (4.13). With this in mind and without any loss of generality we can write,

$$a_0^2 = \frac{m M_{WL}}{V_L^2} \left[B(1 - A) - \frac{m}{M_{WL}} \right], \tag{4.17}$$

and

$$b_0^2 = \frac{mM_{WL}}{V_L^2}\left[B(1 + A) - \frac{m}{M_{WL}}\right]. \tag{4.18}$$

Here M_{WL} is the mass of the charged W-boson of the standard model, and B and A are constants. In place of the two unknowns a_0 and b_0 we now have two other unknowns B and A. We presume that the parameter m is known. Inserting (4.17) and (4.18) into (4.12) and (4.13), we find that

$$m_e^2 = mM_{WL}B(1 - A), \tag{4.19}$$

$$m_\mu^2 = mM_{WL}B(1 + A). \tag{4.20}$$

The mass of the neutrino m is small. This is seen by rearranging (4.19) or (4.20) for m.

5. CALCULATION OF m, a_0 AND b_0

The coupling of the quartet ϕ_0 to the fields $\nu_{eL}(x)$, $\nu_{\mu L}(x)$, $e(x)$, and $\mu(x)$ gives rise to the parameter m of (4.4). This also determines the mass of the neutrinos. Suppose the scalar ϕ_0 is a combination of Z and D bosons with appropriate quantum numbers such that

$$\phi_0 = \begin{pmatrix} (ZD^* + DZ^*) + u_3 & 0 \\ 0 & i(-ZD^* + DZ^*) + v_3 \end{pmatrix}, \tag{5.1}$$

where ZD^* is the scalar formed with the neutral Z-particle of the standard model, and D is the neutral boson of the left–right model. The combination should be built such that it is a scalar and has the other required quantum numbers. We do not go deeply into this matter now because it remains obscure. But (5.1) is enough to extract information. The quartet ϕ_0 is coupled to $\bar{e}e$ or $\bar{\mu}\mu$. In the standard model Z is coupled to $e\gamma_\mu\bar{e}$ through the real $(g_V)_{e\mu}$, where $(g_V)_{e\mu}$ is the vector coupling constant of e or μ with Z (or D) in the neutral sector. Because of (5.1), the parameter m should be proportional to $(g_V)_{e\mu}^2$ because m is real and positive. We therefore note that

$$m \propto (g_V^2)_{e\mu}. \tag{5.2}$$

In an analogous way we assume that the scalars corresponding to the doublets ϕ_L and ϕ_R are proportional to $ZZ^* + V_L$ and $DD^* + V_R$. If this is true, a comparison of the $\bar{e}\gamma_5 e$ coupling constant in (4.5), (4.8), and (4.11) with the real coupling constant $(g_A)_{e\mu}$ of the Z with the axial vector current indicates that

$$a_0^2 \propto (g_A^2)_{e\mu}, \qquad b_0^2 \propto (g_A^2)_{e\mu}. \tag{5.3}$$

From (4.17) and (4.18) we readily observe that very approximately

$$\frac{a_0^2}{b_0^2} \approx \frac{1 - A}{1 + A}. \tag{5.4}$$

To obtain (5.4) we ignored the terms m/M_{WL} in the brackets of (4.17) and (4.18), as m is very small compared to M_{WL}. On the other hand, from (4.19) and (4.20) we have

$$\frac{m_e^2}{m_\mu^2} = \frac{1 - A}{1 + A}. \tag{5.5}$$

A comparison of (5.3)–(5.5) shows that very approximately

$$m_e^2 \propto (g_A^2)_{e\mu}, \qquad m_\mu^2 \propto (g_A^2)_{e\mu}, \tag{5.6}$$

and hence

$$m_e^2 + m_\mu^2 \propto (g_A^2)_{e\mu}. \tag{5.7}$$

Moreover, when we multiply (4.19) and (4.20) we observe that $m_e^2 m_\mu^2 \propto m^2$, and this shows that

$$m_e^2 m_\mu^2 \propto (g_V^2)_{e\mu}^2, \tag{5.8}$$

$$\frac{m_e m_\mu}{m_e^2 + m_\mu^2} \propto \left(\frac{g_V^2}{g_A^2}\right), \tag{5.9}$$

where g_V and g_A are the vector and axial vector coupling constants of the electron (or muon) to the Z particle or D particle. When we add (4.19) and (4.20) we get $2mM_{WL}B$. There is no loss of generality if we arrange this numerical factor 2 to cancel from the numerator and denominator of (5.9), and hence

$$\frac{2m_e m_\mu}{m_e^2 + m_\mu^2} = K\left(\frac{g_V^2}{g_A^2}\right)_{e\mu}. \tag{5.10}$$

In (5.10) K is a proportionality constant. It appears that (5.10) is more fundamental than the arguments that went to establish it. As a first approximation we set $K = 1$; then

$$\frac{2m_e m_\mu}{m_e^2 + m_\mu^2} = \left(\frac{g_V^2}{g_A^2}\right)_{e\mu}. \tag{5.11}$$

Using (4.19) and (4.20) on the LHS of (5.11), we immediately find that

$$(1 - A^2)^{1/2} = \left(\frac{g_V}{g_A}\right)^2_{e\mu}, \tag{5.12}$$

$$A = \left[1 - \left(\frac{g_V}{g_A}\right)^4_{e\mu}\right]^{1/2}. \tag{5.13}$$

Using (5.13) in (4.19), we have

$$m_e^2 = mM_{WL}B\left\{1 - \left[1 - \left(\frac{g_V}{g_A}\right)^4_{e\mu}\right]^{1/2}\right\}. \tag{5.14}$$

The above equation can also be written as,

$$m_e^2 = mM_{WL}\frac{B}{2(g_V^4/g_A^2)}[(g_A^2 + g_V^2)^{1/2} - (g_A^2 - g_V^2)^{1/2}]^2. \tag{5.15}$$

If we set,

$$B = \frac{(g_V/g_A)^4_{\nu e}}{(g_V/g_A)^4_{e\mu}}, \tag{5.16}$$

the masses of the electron and muon are given by

$$m_e^2 = mM_{WL}\frac{(g_V/g_A)^4_{\nu e}}{(g_V/g_A)^4_{e\mu}}\left\{1 - \left[1 - \left(\frac{g_V}{g_A}\right)^4_{e\mu}\right]^{1/2}\right\}, \tag{5.17}$$

$$m_\mu^2 = mM_{WL}\frac{(g_V/g_A)^4_{\nu e}}{(g_V/g_A)^4_{e\mu}}\left\{1 + \left[1 - \left(\frac{g_V}{g_A}\right)^4_{e\mu}\right]^{1/2}\right\}. \tag{5.18}$$

Equations (5.14) and (5.15) are special cases of (5.17). In the above, $(g_V/g_A)^4_{\nu e} = 1$, where $(g_V)_{\nu e}$ and $(g_A)_{\nu e}$ are the vector and axial vector coupling constants of the neutrinos with the neutral Z or D bosons. The ratio $(g_V/g_A)^4_{\nu e}$ is equal to 1 if the neutrino is strictly left-handed. This is true in our case because these are Majorana neutrinos. In selecting the expression (5.16) for B, we wanted to generalize the above formulas to quarks as well (Raju, 1986a).

With the known values of m_e and m_μ we can find $\sin^2\theta_w$ since $(g_V/g_A)^2_{e\mu} = (-1 + 4\sin^2\theta_w)^2$. Here $x_L = \sin^2\theta_w$ is the Weinberg mixing parameter. When we insert m_e^2 and m_μ^2 into (5.11) and treat x_L as an unknown, we get a quadratic equation whose roots are,

$$x_L = 0.2254 \quad \text{or} \quad 0.2746. \tag{5.19}$$

The first value agrees very well with the world average of $x_L \approx 0.23$. The second value will be interpreted here as $x_R = e^2/g_R^2$, which appears in a gauge model based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ and g_R is the gauge constant corresponding to $SU(2)_R$. It should be noted that the sum $x_L + x_R = 0.5$ exactly in (5.19). It is the sum of the roots of the quadratic equation obtained from (5.11).

The neutral sector of the left–right $SU(2)_L \times SU(2)_R \times U(1)$ gauge model contains two neutral bosons Z and D. The weak part of the neutral interaction is given by (Raju, 1985)

$$H_{\text{weak}}^{\text{int}} = g_z J_{ZL} Z + g_z(\beta J_{ZL} - (\alpha + \beta)J_{ZL})D, \tag{5.20}$$

$$J_{ZL} = j_{3L} - x_L j_{em}, \qquad J_{ZR} = j_{3R} - x_R j_{em},$$

$$g_z = \frac{e}{x_L^{1/2}(1 - x_L)^{1/2}},$$

$$\beta = \frac{(x_L x_R)^{1/2}}{(1 - x_L - x_R)^{1/2}},$$

$$\alpha + \beta = \frac{x_L^{1/2}(1 - x_L)}{x_R^{1/2}(1 - x_L - x_R)^{1/2}}. \tag{5.21}$$

The Z and D neutral bosons are mass eigenstates. If the neutrinos are strictly left-handed, then $(g_V/g_A)^2_{\nu,Z} = (g_V/g_A)^2_{\nu,D} = 1$. On the other hand, $(g_V/g_A)_{e\mu}$ of Z or D are not necessarily equal. To have unique masses for the electron and muon we require that

$$(g_V/g_A)^2_{e\mu,Z} = (g_V/g_A)^2_{e\mu,D}. \tag{5.22}$$

Equation (5.22) is equivalent to the following two relations:

$$(g_V/g_A)_{e\mu,Z} = \pm(g_V/g_A)_{e\mu,D}. \tag{5.23}$$

The above relation with the positive sign yields,

$$(1 - 4x_L) = \frac{(\alpha + \beta)4x_R - 4\beta x_L - \alpha}{\alpha + 2\beta}, \tag{5.24}$$

and hence

$$x_L + x_R = 0.5. \tag{5.25}$$

From a world average $x_L = 0.23$ and therefore $x_R = 0.27$. These two roots are also obtained from (5.11). If we use the second of (5.23) with the negative sign, we get

$$x_R = \frac{2}{3}\frac{(1 - x_L)x_L}{(1 - 2x_L)} \tag{5.26}$$

In the above when we use $x_L = 0.23$ it does not lead to an $x_R = 0.27$ and we have no idea whatsoever how to interpret (5.26). The Majorana mass of the neutrinos is given by,

$$m = \frac{m_e m_\mu}{M_{WL}} \left(\frac{g_V}{g_A}\right)^2_{e\mu} \approx 6.5 \text{ eV}, \tag{5.27}$$

where $M_{WL} = 80$ GeV and $x_L = 0.2254$ are used. The neutrinos ν_{eL} and $\nu_{\mu L}$ are Majorana particles with equal mass and opposite *CP* parities. For the τ lepton and its neutrino we have,

$$m_\tau^2 = m_{\nu\tau} M_{WL} \left\{1 + \left[1 - \left(\frac{g_V}{g_A}\right)^4_\tau\right]^{1/2}\right\}, \tag{5.28}$$

where the mass of the τ neutrino $m_\nu = 25$ MeV, if $m_\tau = 2$GeV. Here the constant B is set equal to 1. The τ neutrino does not fit into the above analysis.

6. QUARK MASSES

The analysis carried out appears to be quite general. In (5.17) the product mM_{WL} involves the neutrino mass, whose charge is different from the electron charge. Like the neutrinos, there are two quarks d and u, which have almost equal constituent masses. These are charged particles and hence they cannot have only Majorana mass. However, if (5.17) is any guide, we can assert that,

$$m_c^2 = m_d M_{WL} \frac{(g_V/g_A)^4_{dsb}}{(g_V/g_A)^4_{uct}} \left\{1 - \left[1 - \left(\frac{g_V}{g_A}\right)^4_{uct}\right]^{1/2}\right\}, \tag{6.1}$$

$$m_s^2 = m_u M_{WL} \frac{(g_V/g_A)^4_{uct}}{(g_V/g_A)^4_{dsb}} \left\{1 - \left[1 - \left(\frac{g_V}{g_A}\right)^4_{dsb}\right]^{1/2}\right\}. \tag{6.2}$$

In (6.1) and (6.2)

$$\left(\frac{g_V}{g_A}\right)^2_d = \left(\frac{g_V}{g_A}\right)^2_b = \left(\frac{g_V}{g_A}\right)^2_s = \left(-1 + \frac{4}{3} x_L\right)^2,$$

$$\left(\frac{g_V}{g_A}\right)^2_u = \left(\frac{g_V}{g_A}\right)^2_c = \left(\frac{g_V}{g_A}\right)^2_t = \left(-1 + \frac{8}{3} x_L\right)^2.$$

Here g_V and g_A are the vector and axial vector coupling constants of the respective particles (indicated by the subscripts) with the Z boson. From conventional wisdom if we assume that $m_d \approx m_u = 0.3$ GeV, and if $M_{WL} =$ 80 GeV, then for $x_L = 0.2254$, we have,

$$m_c = 1.7 \text{ GeV} \quad \text{and} \quad m_s = 0.57 \text{ GeV}. \tag{6.3}$$

The Cabibbo angle is given by (Raju, 1986a–c, 1987),

$$\theta_c = \theta_2 - \theta_1. \tag{6.4}$$

Where

$$\tan\theta_2 = \sqrt{\frac{m_d}{m_s}} \quad \text{and} \quad \tan\theta_1 = \sqrt{\frac{m_u}{m_c}}. \tag{6.5}$$

From the above values of the masses we note that

$$\theta_c = 13°11', \tag{6.6}$$

which is an excellent result. The masses of the heavy quarks t and b are not given by expressions similar to (5.18). But all our predictability rests on (5.11). By a mere change of this expression we find that

$$\frac{2m_c m_t}{m_c^2 + m_t^2} = \left(\frac{g_V}{g_A}\right)^4_{uct}, \tag{6.7}$$

$$\frac{2m_s m_b}{m_s^2 + m_b^2} = \left(\frac{g_V}{g_A}\right)^4_{dsb}. \tag{6.8}$$

The similarity of these expressions to (5.11) is striking and therefore the constants g_V and g_A play a vital role in determining the masses of all fermions. From (6.7) and (6.8) we note that for $x_L = 0.2254$,

$$m_t = 134.23 \text{ GeV} \quad \text{and} \quad m_b = 4.69 \text{ GeV}. \tag{6.9}$$

Indeed these values are quite encouraging (Schwarzschild, 1987). Approximate expressions like (6.1) and (6.2) can also be found for the masses of t and b quarks from (6.1), (6.2), (6.7), and (6.0). Thus we have

$$m_t \approx 2\sqrt{m_d M_{WL}}\,\frac{(g_V/g_A)^2_{dsb}}{(g_V/g_A)^6_{uct}}\left\{1 - \left[1 - \left(\frac{g_V}{g_A}\right)^4_{uct}\right]^{1/2}\right\}^{1/2}. \tag{6.10}$$

The above expression can be approximated to,

$$m_t^2 \approx 2m_d M_{WL}\,\frac{(g_V/g_A)^4_{dsb}}{(g_V/g_A)^8_{uct}}. \tag{6.11}$$

in a similar way for the b quark we find that,

$$m_b \approx 2\sqrt{m_u M_{\mathrm{WL}}}\,\frac{(g_{\mathrm{V}}/g_{\mathrm{A}})^2_{uct}}{(g_{\mathrm{V}}/g_{\mathrm{A}})^6_{dsb}}\left[1-\left\{1-\left(\frac{g_V}{g_{\mathrm{A}}}\right)^4_{dsb}\right\}^{1/2}\right]^{1/2}, \qquad (6.12)$$

and

$$m_b^2 \approx 2m_u M_{\mathrm{WL}}\,\frac{(g_{\mathrm{V}}/g_{\mathrm{A}})^4_{uct}}{(g_{\mathrm{V}}/g_{\mathrm{A}})^8_{db}}. \qquad (6.13)$$

In one stretch, we have shown that the masses of all the known fermions are closely linked to their vector and axial vector coupling constants with the Z boson.

7. SUMMARY

We have found that the left-handed neutrinos $\nu_{e\mathrm{L}}$ and $\nu_{\mu\mathrm{L}}$ are Majorana neutrinos with equal mass and with opposite *CP* parities. The existing experimental data are not incompatible with the assumption that to the charged leptons (e.g., e and μ) there corresponds one four-component neutrino whose LH and RH components enter into the weak lepton currents.

Using only the masses of the electron and muon, we have determined the Weinberg mixing parameters x_{L} and x_{R} and the mass of their neutrinos. With the help of m_u and m_d we found the constituent masses of m_c, m_t, m_b and m_s quarks, all of which have the expected values. In addition, in one stretch we have shown that the masses of all fermions are intimately connected with vector and axial vector coupling constants of the fermions with the Z (or D) bosons. We found only one set of the masses of quarks with $x_{\mathrm{L}} = 0.2254$. The Cabibbo angle computed here turns out to be rather exact. With the values of the masses computed here, the exact KM matrix can be evaluated. In the case of all the fermions the masses are determined by $(g_{\mathrm{V}}/g_{\mathrm{A}})^2$ ratios or higher powers of it. Until now no serious attention has been paid to this connection. When we wanted to bring in the $(g_{\mathrm{V}}/g_{\mathrm{A}})^2$ connection to the masses, we were led to the idea that the Higgs scalars are bound states of neutral Z and D bosons with appropriate quantum numbers. When a spin-one particle is combined with another spin-one particle, we will have 2,1,0 spin particles. We used only the scalar particles. The fate of the remaining ones needs to be studied seriously and accounted for. This probably leads to more exciting physics. Incidentally, the mass of the standard Higgs scalar ϕ_{L} must be around $2m_z = 180$ GeV, as it is a bound state of Z and $Z^\star$ (Osland and Wu, 1992). The two Majorana masses of the neutrinos are equal and hence they correspond to a single four-component field $\nu(x)$ which has a

Dirac mass m, and the Majorana neutrinos have opposite CP parities. The tau-neutrino problem is a separate one and it remains to be solved.

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REFERENCES

Bailen, D., and Love, A. (1974). *Nuclear Physics B*, **69**, 142
Bilenkey, S. M., and Petcov, S. T. (1987). *Reviews of Modern Physics*, **59**, 671.
Konopinsky, E. J., and Mahmoud, H. (1953). *Physical Review*, **92**, 1045.
Marshak, R. E., and Sudarshan, E. C. G. (1958). *Physical Review*, **109**, 1860.
Mohapatra, R. N. (1981). *Physical Review D*, **23**, 165.
Mohapatra, R. N., and Senjanovic, G. (1980). *Physical Review Letters*, **48**, 912.
Osland, P., and Wu. T. T. (1992). CERN-TH 6557.
Raju, C. (1985). *Pramana*, **24**, L657.
Raju, C. (1986a). *Chechoslovak Journal of Physics*, **12**, 1350.
Raju, C. (1986b). *International Journal of Theoretical Physics*, **25**, 3739.
Raju, C. (1986c). *International Journal of Theoretical Physics*, **25**, 1105.
Raju, C. (1987). *International Journal of Theoretical Physics*, **27**, 551.
Schwarzschild, B. (1985). *Physical Today*, **17**, 111
Weinberg, S. (1967). *Physical Review Letters*, **19**, 1264.
Zeldovich, Y. B. (1952). *Doklady Akademii Nauk SSSR*, **86**, 505.